

Nucleon tensor form factors in a relativistic confined quark model

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We present results for the isotriplet and isosinglet tensor form factors of the nucleon in the relativistic confined quark model. The model allows us to calculate not only their normalizations at $Q^2 = 0$ and the related tensor charges, but also the full Q^2 -dependence. Our results are compared to existing data and predictions of other theoretical approaches. We stress the importance of these form factors for the phenomenology of physics beyond the Standard Model.

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I. INTRODUCTION

Study of nucleon structure is one of the promising tools to understand hadronic matter and phenomenological aspects of strong interactions. From a modern point of view, all information about the five-dimensional structure of the nucleon (here we count one longitudinal coordinate x , two coordinates in transverse impact space \mathbf{b}_\perp and two coordinates in transverse momentum space \mathbf{k}_\perp) [1–3] is given by the QCD Wigner distributions for quarks in the nucleon $W(x, \mathbf{b}_\perp, \mathbf{k}_\perp)$. At leading twist $\tau = 2$ there are $16 = 4 \times 4$ Wigner distributions [4], since the nucleon and the quarks could each be in four polarization states — unpolarized or polarized along three orthogonal directions. Integrating or Fourier transforming, the Wigner distributions, one can get all known nucleon quantities: distributions, form factors, charges. In particular, integrating $W(x, \mathbf{b}_\perp, \mathbf{k}_\perp)$ over \mathbf{k}_\perp and Fourier transforming with respect to \mathbf{b}_\perp gives three-dimensional images of nucleon — eight generalized parton distribution (GPDs) $H(x, \xi, t)$, where ξ is the skewness and $t = q^2$ is the squared momentum transfer. Integrating $W(x, \mathbf{b}_\perp, \mathbf{k}_\perp)$ over \mathbf{b}_\perp produces eight transverse momentum densities (TMDs) $f(x, \mathbf{k}_\perp)$. Further integration of the TMDs over \mathbf{k}_\perp gives the one-dimensional parton distribution functions $f(x)$, while integration of the GPDs over the longitudinal variable x gives eight elastic form factors $F(t)$: four of them correspond to the matrix elements of the vector and axial current and the other four to the matrix element of the tensor current. One of the tensor form factors vanishes due to T -invariance. Including T -invariance, one therefore has seven form factors: two vector, two axial and three tensor form factors.

The scope of the present paper is to calculate the

three tensor nucleon form factors in the covariant confined quark model, proposed and developed in Refs. [5]. The model aims at an unified relativistic description of the bound state structures of hadrons and exotic states.

Following the pioneer paper of [6], the matrix element of the tensor current $J_f^{\mu\nu} = \bar{q}\sigma^{\mu\nu}\tau_f q$ between nucleon states can be written in terms of three dimensionless, invariant form factors T_i^f ($i = 1, 2, 3$ and $f = 0, 3$),

$$\begin{aligned} \langle N(p_2) | J_f^{\mu\nu} | N(p_1) \rangle &= \bar{u}_N(p_2) \tau_f \left[\sigma^{\mu\nu} T_1^f(q^2) \right. \\ &+ \frac{i}{m_N} (q^\mu \gamma^\nu - q^\nu \gamma^\mu) T_2^f(q^2) \\ &+ \left. \frac{i}{m_N^2} (q^\mu P^\nu - q^\nu P^\mu) T_3^f(q^2) \right], \quad (1) \end{aligned}$$

where p_1 and p_2 are the momenta of the initial and final nucleon, $q = p_1 - p_2$ is the momentum transfer variable and $P = p_1 + p_2$. Here, $\sigma^{\mu\nu} = i/2[\gamma^\mu, \gamma^\nu]$ is the anti-symmetric spin tensor matrix; $\tau_0 \equiv I = \text{diag}(1, 1)$ and $\tau_3 = \text{diag}(1, -1)$ are the isospin matrices.

This set of form factors is related to the chiral-odd spin-dependent generalized parton distributions (GPDs) [7], $H_T^q(x, \xi, q^2)$, $E_T^q(x, \xi, q^2)$ and $\tilde{H}_T^q(x, \xi, q^2)$, by taking the first moments of latter over the longitudi-

nal variable x :

$$\begin{aligned} \int_{-1}^1 dx H_T^q(x, \xi, q^2) &= H_T^q(q^2) \equiv T_1^q(q^2), \\ \int_{-1}^1 dx E_T^q(x, \xi, q^2) &= E_T^q(q^2) \equiv 2T_2^q(q^2), \\ \int_{-1}^1 dx \tilde{H}_T^q(x, \xi, q^2) &= \tilde{H}_T^q(q^2) \equiv 2T_3^q(q^2). \end{aligned} \quad (2)$$

Equation (2) gives the relations between the Adler set of tensor form factors ($T_i^u = (T_i^0 + T_i^3)/2$, $T_i^d = (T_i^0 - T_i^3)/2$) and the more popular set used nowadays: H_T^q , E_T^q , and \tilde{H}_T^q . As we point out before, chiral-odd GPDs together with chiral-even GPDs encode information about quark structure of the nucleon (the current status of the field is recently reviewed in [8], which are subject of extensive experimental study [from first measurements at DESY (HERA Collaboration) and JLab (CLAS Collaboration) [9, 10] in the valence quark region to a comprehensive study at CERN (COMPASS Collaboration) [11] in the small- x sea quark and gluon region] and various theoretical analysis.). Next, the progress in the study of the GPDs have been done in many papers. For example, for a status report of the momentum-dependent calculations, see Ref. [12] (chiral-even GPDs) and Refs. [13, 14] (chiral-odd GPDs).

The normalizations of the T_1^0 and T_1^3 form factors represent the conventional nucleon tensor charges (g_N^3 , g_N^0). They are related to the tensor charges of quarks δq ($q = u, d$) in the proton so that [6]:

$$\begin{aligned} \delta u &= \frac{T_1^0(0) + T_1^3(0)}{2}, \quad \delta d = \frac{T_1^0(0) - T_1^3(0)}{2}, \\ g_N^3 &= \delta u - \delta d = T_1^3(0), \quad g_N^0 = \delta u + \delta d = T_1^0(0). \end{aligned} \quad (3)$$

The tensor charge of the quarks measures the light-front distribution of transversely polarized quarks inside a transversely polarized proton [15]:

$$\delta q = \int_0^1 dx h_1^q(x), \quad (4)$$

where $h_1^q(x)$ is the transversity distribution of the valence quarks of the flavor q in the nucleon. In Ref. [16] the transversity distributions were extracted from the combined data of the Belle, COMPASS and HERMES Collaborations. The corresponding values of the quark tensor charges in the proton are [16]

$$\delta u = 0.39^{+0.18}_{-0.12}, \quad \delta d = -0.25^{+0.30}_{-0.10}. \quad (5)$$

In the near future, significantly more accurate measurements of the nucleon/quark tensor charges are expected

at JLab by the two collaborations SoLID (Hall A) and CLAS12 (Hall B).

In Ref. [17] the valence quark contributions to the nucleon tensor charge were estimated based on a global analysis of the Collings azimuthal asymmetries in e^+e^- annihilation and SIDIS processes with the full QCD dynamics and including the appropriate transverse momentum-dependent (TMD) evolution effects at next-to-leading logarithmic (NLL) order and perturbative corrections at next-to-leading order (NLO):

$$\delta u = 0.39^{+0.16}_{-0.20}, \quad \delta d = -0.22^{+0.31}_{-0.10}. \quad (6)$$

One obtains the following estimates for the isovector and isoscalar nucleon charge:

$$g_N^3 = 0.61^{+0.26}_{-0.51}, \quad g_N^0 = 0.17^{+0.47}_{-0.30}. \quad (7)$$

In Refs.[18, 19] the u and d quark valence contributions to the tensor charges have been evaluated using experimental data on di-hadron production from the the HERMES, COMPASS, and BELLE Collaborations. The impact of recent developments in hadron phenomenology on extracting possible fundamental tensor interactions beyond the Standard Model has been studied in a recent paper [20].

The tensor nucleon charges and the related $T_1^{0,3}(0)$ values have been calculated in the literature within various theoretical frameworks: QCD sum rules approach [21–24], Lattice QCD [25]–[32], chiral soliton model (CSM) [33, 34], chiral chromomagnetic model (CCM) [35], spectator model (SM) [36], light-front quark model (LFQM) [37, 38], model-independent sum rules [39], and Dyson-Schwinger equation (DSE) approach [40, 41]. To the best of our knowledge, only Ref. [6] addressed the calculation of all the nucleon tensor form factors $T_{1,2,3}^f(0)$ in the frameworks of the quark model (QM) and MIT bag models.

The quark tensor charges δq has implications for the CP violation phenomena like the neutron electric dipole moment (EDM) in beyond Standard Model (SM) theories. In particular, the neutron EDM is given by linear combination of the quark EDMs d_q [42, 43],

$$d_n = \sum_{q=u,d,s} \delta q d_q. \quad (8)$$

Therefore, an improved knowledge of quark tensor charges could give an opportunity to get more stringent constraints on the parameters of CP violation d_q .

On the other hand, the momentum dependence of the nucleon tensor form factors $T_{1,2,3}^f(q^2)$ is of great importance for some rare processes beyond the SM with the participation of nucleons. This is particularly true for the neutrinoless double beta ($0\nu\beta\beta$) decay violating the total lepton number by two units and forbidden in the SM. The short-range mechanisms of $0\nu\beta\beta$ -decay, originating from a heavy particle exchange [44, 45], involve the tensor nucleon form factors for space-like transferred

momenta $Q^2 = -q^2$. It is well known that for various high-scale models [46] the tensor contribution dominates the $0\nu\beta\beta$ -decay [45]. This fact shows the importance of a reliable knowledge of the $T_{1,2,3}^f(q^2)$ for space-like transferred momenta. So far for estimates of the tensor contribution in $0\nu\beta\beta$ -decay only, the results of the above mentioned Ref. [6] have been used. However, since the publication of Ref. [6], significant progress has been made in the understanding of nucleon structure. These circumstances motivated us to carry out the computation of the nucleon tensor form factors in the covariant confined quark model, which correctly reproduces various hadronic properties and has proved to be a framework with a significant predictive power [5].

The paper is organized as follows. Section II outlines the formal key aspects of the covariant confined quark model. In Sec. III we present and discuss the model predictions for the tensor form factors of nucleons. We present a comparison of our results with predictions of other approaches.

II. FORMALISM

In the covariant confined quark model (CCQM) [5], the nucleons N are coupled to their constituent quarks according to the Lagrangian [5, 47]:

$$\begin{aligned}\mathcal{L}_{\text{int}}^N &= g_N \bar{N}(x) \cdot J_N(x) + \text{H.c.}, \\ J_N(x) &= (1 - x_N)J_N^V + x_N J_N^T,\end{aligned}\quad (9)$$

where g_N is the coupling constant. Here, J_N^V and J_N^T are the vector and tensor interpolating three-quark currents with the quantum numbers of the nucleon N ,

$$\begin{aligned}J_p^V(x) &= \int dx_1 \int dx_2 \int dx_3 F_N(x; x_1, x_2, x_3) \\ &\quad \times \epsilon^{a_1 a_2 a_3} \gamma^\mu \gamma^5 d^{a_1}(x_1) u^{a_2}(x_2) C \gamma_\mu u^{a_3}(x_3), \\ J_p^T(x) &= \int dx_1 \int dx_2 \int dx_3 F_N(x; x_1, x_2, x_3) \\ &\quad \times \epsilon^{a_1 a_2 a_3} \sigma^{\mu\nu} \gamma^5 d^{a_1}(x_1) u^{a_2}(x_2) C \sigma_{\mu\nu} u^{a_3}(x_3), \\ J_n^V(x) &= - \int dx_1 \int dx_2 \int dx_3 F_N(x; x_1, x_2, x_3) \\ &\quad \times \epsilon^{a_1 a_2 a_3} \gamma^\mu \gamma^5 u^{a_1}(x_1) d^{a_2}(x_2) C \gamma_\mu d^{a_3}(x_3), \\ J_n^T(x) &= - \int dx_1 \int dx_2 \int dx_3 F_N(x; x_1, x_2, x_3) \\ &\quad \times \epsilon^{a_1 a_2 a_3} \sigma^{\mu\nu} \gamma^5 u^{a_1}(x_1) d^{a_2}(x_2) C \sigma_{\mu\nu} d^{a_3}(x_3),\end{aligned}$$

and x_N is the mixing coefficient of the J_N^V and J_N^T currents. Here, a_i are the color indices and $C = \gamma^0 \gamma^2$ is the charge conjugation matrix.

In the CCQM the nonlocal vertex function F_N characterizes the finite size of the nucleon. Replacing the Cartesian coordinates x_i of the constituent quarks with their Jacobi and center mass coordinates, one can relate F_N [47] by a Fourier transformation to a momentum-space

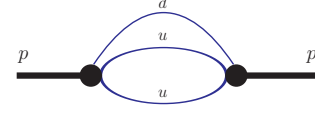


FIG. 1: The proton mass operator.

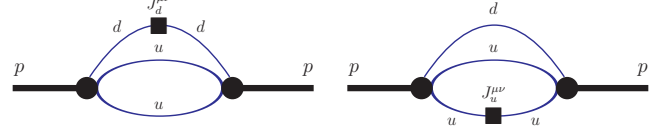


FIG. 2: Diagrams contributing to the proton tensor form factors.

vertex function $\bar{\Phi}_N(-P^2)$. As motivated in Ref. [47], a reasonable choice for this function is a Gaussian form

$$\bar{\Phi}_N(-P^2) = \exp(P^2/\Lambda_N^2), \quad (10)$$

where Λ_N is a size parameter describing the distribution of quarks inside the nucleon N . The values for these parameters were fixed in Refs. [47–51] from the analysis of various hadronic observables. It is worth noting that the Minkowskian momentum variable P^2 turns into the Euclidean form $-P_E^2$ needed for the appropriate fall-off of the correlation function (10) in the Euclidean region. For given values of the size parameters Λ_N , the coupling constant g_N is determined by the compositeness condition suggested by Weinberg [52] and Salam [53] (for a review, see [54]), which is one of the key elements of our approach (for further details, see [55]). The compositeness condition implies that the renormalization constant of the hadron wave function is set equal to zero:

$$Z_N = 1 - \Sigma'_N = 0, \quad (11)$$

where Σ'_N is the on-shell momentum derivative of the proton mass operator Σ_H . The compositeness condition guarantees the correct charge normalization for a charged bound state (see e.g. [48]). In Fig. 1 we show the diagram describing the proton mass operator in the CCQM.

The calculation of matrix elements of baryonic transitions in the CCQM has been discussed in detail in Refs. [47, 49, 50]. The matrix element of the tensor current is described by two-loop Feynman-type diagrams involving nonlocal vertex functions. These diagrams for the case of proton are shown in Fig. 2 and correspond to the insertion of the operators $J_d^{\mu\nu} = \bar{d}\sigma^{\mu\nu}d$ and $J_u^{\mu\nu} = \bar{u}\sigma^{\mu\nu}u$ from the left to the right, respectively. For the neutron it is sufficient to replace $u \leftrightarrow d$. In the calculation of the quark-loop diagrams in Figs. 1 and 2, we use model parameters determined in our previous studies [47, 48, 51]: $m_u = m_d = 235$ MeV (the u, d constituent quark mass), $\lambda = 181$ MeV (an infrared cutoff parameter responsible for the quark confinement), $x_N = 0.8$ (the nucleon cur-

rent mixing parameter), $\Lambda_N = 500$ MeV (the nucleon size parameter).

One of the notable advantages of our approach is that we are able to reproduce data on meson and baryon properties with the same (universal) masses of the constituent quarks. Quark confinement is guaranteed in our model and described by an universal confinement scale $\lambda = 181$ MeV both in the meson and baryon sectors. The parameter λ is an infrared cutoff parameter, which defines the upper limit in the integration of quark-loop integrals over scale parameter. This parameter λ is of order of light quark mass $\lambda \sim m_u = m_d$, and it is the scale in our approach. Such a choice is consistent with fixing the scale in the approaches based on the solution of Bethe-

Salpeter equations (see detailed discussion in Ref. [41]).

TABLE I: Parameters of the double-pole approximation of the nucleon tensor form factors in Eqs. (12).

	T_1^0	T_2^0	T_3^0	T_1^3	T_2^3	T_3^3
$F(0)$	0.761	- 1.569	- 0.722	1.255	- 3.028	0.988
a	1.396	1.940	1.518	1.195	1.766	1.778
b	1.479	1.941	1.109	0.548	1.410	1.443

III. RESULTS AND DISCUSSIONS

The Q^2 -dependence of the tensor of nucleon form factors have been calculated before in QCD SR [23, 24] for values of Q^2 larger than 1 GeV². The results of our numerical two-loop calculation of the nucleon tensor form factors are presented in Figs. 3-5, where we show their Q^2 -dependence up to 1 GeV². In Fig. 3 we compare our results for the $T_1^0(Q^2)$ and $T_1^3(Q^2)$ form factors with results of Lattice QCD [26] and the CSM approach [34]. One can see that our results are close to the predictions of the CSM approach [34], and both approaches produce curves, lower than the corresponding Lattice QCD predictions [26]. Our curves are very accurately approximated by a double-pole parametrization of the kind

$$F(Q^2) = \frac{F(0)}{1 - as + bs^2}, \quad s = \frac{Q^2}{m_N^2}, \quad (12)$$

where $F = T_{1,2,3}^f$. The numerical values of the parameters in Eq. (12) are shown in Table I.

Following idea of Ref. [39] to summarize results for the quark and nucleon tensor charges in Table II, we present our predictions for those quantities in comparison with the results of other theoretical approaches and the data analysis. For completeness we present the predictions of the nonrelativistic SU(6) quark model and LFQM in the ultrarelativistic limit [37, 38]. As usual different approaches present their predictions at different scales. One can see that our predictions are in a reasonable agreement with the results of other theoretical approaches. In particular, our result for the $\delta u = 1.008$ is close to the central values of the lattice QCD predictions [25]-[32], while our result for $\delta d = -0.247$ is in very good agreement with lattice QCD giving δd in the range from -0.236 to -0.23 . Also one should stress that our prediction for δu is bigger than the corresponding quantity extracted from data analysis [17, 19].

In Table III we compare our results for the normalizations of the tensor form factors with the predictions of

the quark model and MIT bag model [6]. As seen from Table III, our results for the normalization values of the form factors are closer to the MIT bag model than to the nonrelativistic quark model. Also we compare our predictions for the tensor anomalous magnetic moments $\kappa_q^T = -2(T_2^q(0) + 2T_3^q(0))$ with results of Ref. [13]. In particular, our results $\kappa_u^T = 4.065$ and $\kappa_d^T = 1.961$ are similar to the results $\kappa_u^T = 3.43 \pm 0.26$ and $\kappa_d^T = 1.37 \pm 0.34$ derived in Ref. [13] at scale 0.8 GeV².

In summary, we have calculated the isosinglet and isotriplet tensor nucleon form factors for the space-like momentum transfer $Q^2 \leq 1$ GeV² in the covariant confined quark model. The calculation of matrix elements of baryonic transitions in the CCQM has been discussed in detail in Refs. [47, 49, 50]. In our approach the matrix element of the tensor current is described by two-loop Feynman-type diagrams involving nonlocal vertex functions. In the calculation of the quark-loop diagrams, we used the parameters fixed in our previous papers [47, 48, 51]: constituent quark masses $m_u = m_d = 235$ MeV, an infrared cutoff parameter $\lambda = 181$ MeV responsible for the quark confinement, the interpolating nucleon current mixing parameter $x_N = 0.8$, and the nucleon size parameter $\Lambda_N = 500$ MeV. We performed a comparison of our results with known results of other approaches. The analytic parametrization (12) of the Q^2 -dependence of the form factors allows their convenient incorporation to the analysis of rare processes involving nucleons, in particular, nuclear structure calculations for neutrinoless double beta decay.

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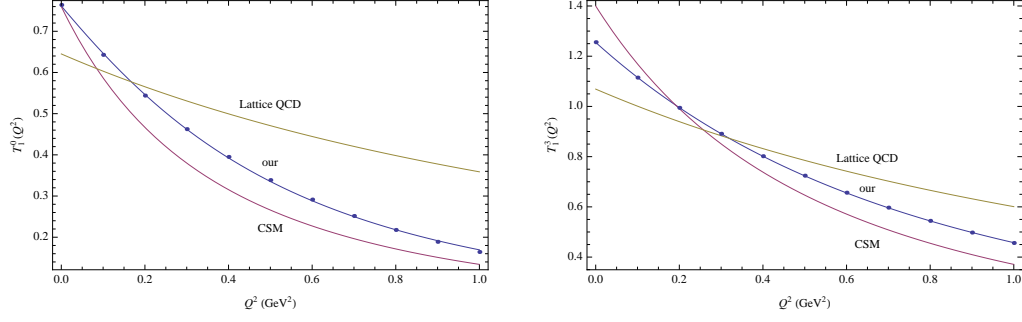


FIG. 3: Form factors $T_1^0(Q^2)$ and $T_1^3(Q^2)$: CSM approach [34] at scale $\mu^2 = 0.36 \text{ GeV}^2$, Lattice QCD [26] at scale $\mu^2 = 4 \text{ GeV}^2$, and our results [solid line (approximation), dots correspond to the exact result] at scale $\mu^2 \sim \lambda^2 = 0.033 \text{ GeV}^2$.

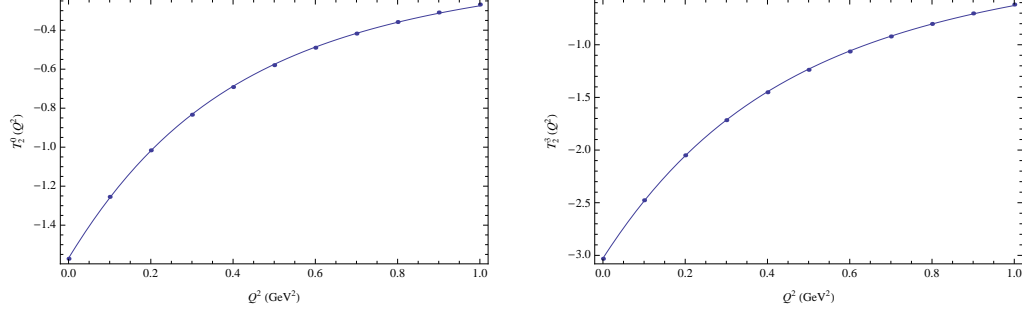


FIG. 4: Form factors $T_2^0(Q^2)$ and $T_2^3(Q^2)$: solid line (approximation), dots correspond to the exact result.

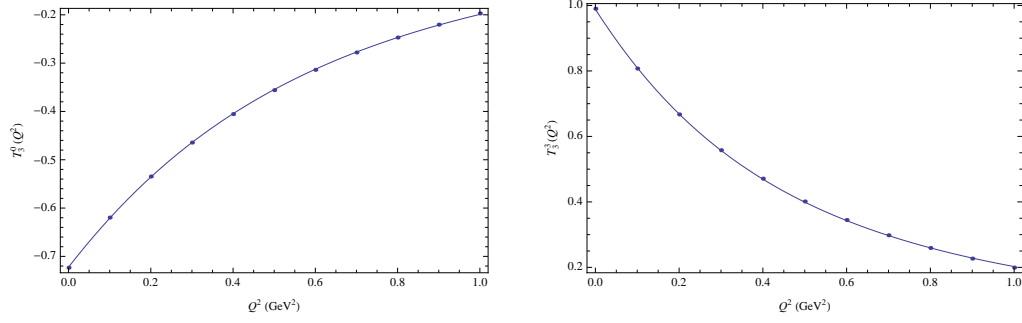


FIG. 5: Form factors $T_3^0(Q^2)$ and $T_3^3(Q^2)$: solid line (approximation), dots correspond to the exact result.

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TABLE II: Comparison of the results for the quark and nucleon tensor charges.

Approach	δu	δd	g_N^3	g_N^0
QM [6]	1.16	- 0.29	1.45	0.87
MIT bag Model [6]	1.105	- 0.275	1.38	0.83
QCD SR [21]	1.33 ± 0.53	0.04 ± 0.02	1.29 ± 0.51	1.37 ± 0.55
Lattice QCD [25]	0.84	- 0.23	1.07	0.61
Lattice QCD [27]			$1.038 \pm 0.011 \pm 0.012$	
Lattice QCD [28]				0.671 ± 0.013
Lattice QCD [30]	0.791 ± 0.053	$- 0.236 \pm 0.033$	1.027 ± 0.062	
Lattice QCD [31]			$1.31 \pm 0.12 \pm 0.11$	$0.81 \pm 0.20 \pm 0.07$
Lattice QCD [32]	0.774 ± 0.066	$- 0.233 \pm 0.028$	1.020 ± 0.076	0.541 ± 0.067
CSM [33]	1.12	- 0.42	1.54	0.70
CSM [34]	1.08	- 0.32	1.40	0.76
CCM [35]	0.969	- 0.250	1.219	0.719
SM [36]	1.218	- 0.255	1.473	0.963
LFQM [38]	1.167	- 0.292	1.458	0.875
Sum rules I [39]	0.965 ± 0.125	$- 0.37 \pm 0.14$	1.335 ± 0.095	0.60 ± 0.09
Sum rules II [39]	1 ± 0.11	$- 0.41 \pm 0.12$	1.42 ± 0.04	0.60 ± 0.09
Data analysis [16]	$0.39^{+0.18}_{-0.12}$	$-0.25^{+0.30}_{-0.10}$	$0.64^{+0.48}_{-0.22}$	$0.14^{+0.58}_{-0.42}$
Data analysis [17]	$0.39^{+0.16}_{-0.20}$	$-0.22^{+0.31}_{-0.10}$	$0.61^{+0.26}_{-0.51}$	$0.17^{+0.47}_{-0.30}$
Data analysis [19]	0.39 ± 0.15	-0.41 ± 0.52		
DSE [40]	0.8	-0.2	1.0	0.6
DSE [41]	0.693 ± 0.10	-0.14 ± 0.02	0.83 ± 0.12	0.55 ± 0.08
Nonrelativistic SU(6) QM	4/3	- 1/3	5/3	1
LFQM (Ultrarelativistic limit) [37, 38]	2/3	- 1/6	5/6	1/2
Our results	1.008	- 0.247	1.255	0.761

TABLE III: Comparison of the results for the normalizations of the nucleon tensor form factors.

Approach	T_1^0	T_2^0	T_3^0	T_1^3	T_2^3	T_3^3
QM [6]	0.87	- 0.88	- 1.44	1.45	- 1.48	0.41
MIT bag Model [6]	0.83	- 1.98	- 0.78	1.38	- 3.30	1.34
Our results	0.761	- 1.569	- 0.722	1.255	- 3.028	0.988

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